

Seriation and Multivariate Statistics

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'The prehistorian shall no longer as his only task describe and compare artifacts from different countries, and investigate the way of life in forgotten times. He now also seek to trace the inner connection between the types, and show how the one has developed from the other. We call this typology. The typological investigation is very simple in its principles. To study an artifact group, you first gather a material as large as possible, *order* it in the way that the inner criterias of the individual types seems to demand, and afterwards investigate if the conditions under which the individual types has been found can support the relative dating you have reached.' (Montelius 1884, p. 1, my translation).

In this way the Swedish archaeologist Oscar Montelius described the typological method that he, and another Swedish archaeologist Hans Hildebrandt, developed in the 1870's and 80's.

Heavily attacked by the Danish archaeologist Sophus Müller (1884), who claimed that the basic method of chronological ordering in archaeology was not the typological method, but a comparison of closed find contexts, Montelius conceded that the ordering was based on the typological method, and the comparisons of closed finds simultaneously (Montelius 1884).

Ever since the days of Montelius and Müller, the basis of a 'true' chronological investigation in Scandinavian archaeology has been the typological method, and the study of associated finds in closed contexts—mostly graves or hoards. But, there never existed formalized techniques associated with these investigations. When Montelius spoke of gathering as much material as possible, he meant it more or less literally. You either gathered the actual material, or you gathered drawings/sketches of it, and then you sorted it, justifying your classification in words.

Not until the 1960's did a Scandinavian archaeologist try to give a formalized description of the typological method, and the study of find combinations. It had then been practised for almost a century in Scandinavia. The description was given by Mats P. Malmer, who throughout his work has been a strong proponent for clarity and explicit definitions in archaeological research.

In the opening chapter *The methodological base of the study of artifacts* in his book on *Problems of methodology in the art history of the Iron Age* (Malmer 1963, my translation), he gave a most influential outline of the typological method, and the study of find associations.

Initially he focused on the description of artifacts, where he strongly advocated a formal description using well defined 'typological elements'. Then, turning to the typological method, he introduced formal criteria of continuity among artefacts. He defined two criteria of continuity based on what he called constant and variable elements. The first would be elements described on a dichotomised nominal scale, while the other would be elements described on an ordinal scale. He defined the criteria of continuity as follows:

'If the types in a typological series are distinguished from each other by constant elements being systematically substituted by other constant

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	A	B	C	D	E	F	G	H	I	J	K	L
1	1	1	1	1	1	0	0	0	0	0	0	0
2	0	1	1	1	1	1	0	0	0	0	0	0
3	0	0	1	1	1	1	1	0	0	0	0	0
4	0	0	0	1	1	1	1	1	0	0	0	0
5	0	0	0	0	1	1	1	1	1	0	0	0
6	0	0	0	0	0	1	1	1	1	1	0	0
7	0	0	0	0	0	0	1	1	1	1	1	0
8	0	0	0	0	0	0	0	1	1	1	1	1

Table 21.1:

elements, then we have the first criteria of continuity. If the types in the series are designated with numbers, and the typological elements with letters, the series may then have the following structure: 1) A + B + C + D + E. 2) B + C + D + E + F. 3) C + D + E + F + G. 4) D + E + F + G + H. The second criteria of continuity is present if one or more variable elements shows variation. In the following series B and E are variable elements, while the others are constant elements: 1) A + B₁ + C + D + E₁. 2) A + B₂ + C + D + E₂. 3) A + B₃ + C + D + E₃. 4) A + B₄ + C + D + E₄. 5) A + B₅ + C + D + E₅. Continuity is established if all the variable elements change in a systematic way.' (Malmer 1963, p. 27, my translation).

Having established the formal structure of the typological method, Malmer turned to the investigation of find contexts. He argued that logically there is no real difference between the approach used on find associations and the one used in the typological method. Indeed, the first criteria of continuity, as outlined above, would also hold true if the numbers represented finds (graves, hoards or what ever), and the letters represented types in those graves.

Malmer goes as far as to suggest that there is no logical difference what so ever between the chronological informations contained in a set of artifacts, and those contained in a set of find contexts. This, however, has been vigorously countered, and rightly so, by Bo Gräslund (Gräslund 1974, p. 27ff). He argues that although the logic of the methods dealing with the two types of information may be the same, the quality of the information contained in a set of artifacts on the one hand, and in a set of find associations on the other, is certainly not the same. The find associations are far the most informative.

Malmers' analysis resulted in a considerable step forward in the understanding of the nature of the traditional methods of Scandinavian archaeology. It did not, however, lead to any progress towards formal techniques associated with these methods. Malmer, who otherwise was very keen on the introduction of 'objective' techniques, made no suggestions, and neither did Gräslund, who in his book on relative chronology (1974), discussed both the traditional methods and the 'American quantitative seriation methods', and commented on the clear parallels between the presence-absence seriation method, and the traditional Scandinavian methods. Today it is no major discovery to realize that Malmers' first criterion of continuity, applying to both the typological method and to the investigation of find associations, can be rewritten in matrix form, as illustrated in Table 21.1. This form is well known from the literature on seriation, representing the ideal structure of a seriated matrix.

During the seventies and eighties seriation techniques have gradually been spreading in Scandinavia, and at last a formal technique has been applied to a method that has been the backbone of Scandinavian archaeology for more than a century.

So far Scandinavian attempts at seriation by matrix reordering have been of the iterative kind, and almost exclusively obtained by hand sorting. A problem with this approach is that one can always obtain some sort of order, looking quite similar to what one is expecting—a concentration along the diagonal. And what is more, it is difficult, not to say impossible, to decide from the reorganised matrix if the obtained order is satisfactory: that is, if you have obtained a result sufficiently close to the ideal, to allow the conclusion that the criteria of continuity has been met with. As Kendall expressed it:

'As long as we work solely with permutations, the method, or any variant of it, will of necessity yield a linear ordering as an answer, and so will be given no opportunity to 'fail'. I attach great importance to methods which are capable of failure, because it is obvious that in some ill-chosen problems a method ought to fail, and thus warn us that we are taking too simple-minded a view of the data.' (Kendall 1971, p. 218).

Kendall suggested a method that could fail. He used the Kruskal non-metric scaling program (MDSCAL) on a set of artificial data that met the criteria for a perfect seriation. The result was a horse shoeshaped or semi-circular linear distribution of the units on the first two principal axes. Used on real data it is possible to decide how well the criteria of continuity has been met with by looking at how well the points follow a semi-circle. Further, one can take the order of the points in the semi circle as an expression of the best order in which to sort the matrix.

However, it is not only non-metric scaling that can be used for this type of seriation. Correspondence analysis (CA) (Greenacre 1981, Greenacre 1984, Hill 1974), and metric scaling as performed through principal coordinate analysis (PCO) (Gower 1966) are also suitable. Both of these analyses are based on orthogonal regression; the former through a 'singular value decomposition' of a rectangular matrix ($n \times m$ where $m \leq n$) and the latter through a 'spectral decomposition' of a quadratic matrix.

The PCO analysis is closely related to MDSCAL, as it takes a similarity coefficient matrix with positive semi definite qualities as its starting point. It is thus a Q-type of analysis working with relations between units. Its immediate advantage compared to MDSCAL is that being a metric method it is considerably cheaper in CPU time.

Because it starts from a similarity coefficient matrix, PCO is very flexible with regard to data described by variables on different scales. Thus PCO based on Gower's coefficient of similarity (Gower 1971) would be able simultaneously to analyse all the types of data included in Malmers' criteria of continuity, including those on an ordinal scale, provided they undergo some initial transformation.

The CA is simultaneously a R-type and a Q-type analysis. Its origin lies with the study of two-dimensional tables of contingency, and its extension to deal with multivariate data is also restricted to categorical data. The CA method thus has clear limits with respect to the types of data it can handle, being counts and presence-absence data, and due to the special weighting based on column and row marginal sums that takes place in CA, the two types of data cannot be mixed in the same analysis. CA can therefore handle the situation described in Malmers' first criteria of continuity only, but due to its simultaneous R- and Q- type function, and partly due to the way the data is weighted, it is a very powerful method that is preferable whenever applicable.

To test the way CA and PCO deal with data which are ideally distributed with reference to Malmers' first criteria of continuity, I have analysed a number of 50 by 50 matrices displaying different variations of the basic pattern shown in Table 21.1. The results from each of the two methods are the same regardless of the variation in the matrices analysed.

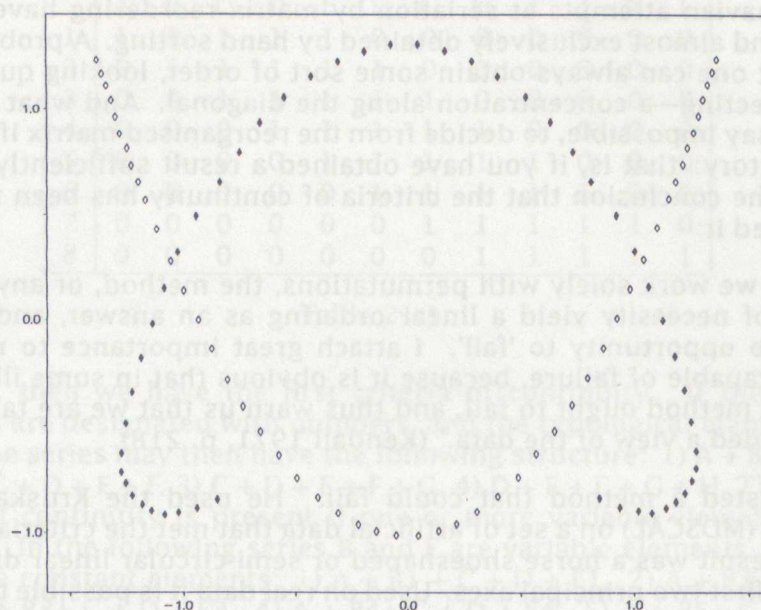


Figure 21.1: Plot of the first two principal axes from a PCO (heavy signatures) and a CA (open signatures) of a 50 by 50 matrix containing ideal data. After Madsen 1988b, Figure 11.

The first two axes in a PCO thus place the units in a formation, reminiscent of Kendall's horseshoe (Fig. 21.1). To understand why this formation is created, one has to remember that PCO is based on similarity coefficients between the units. Viewed against this background the first two principal axes can be seen as a two-dimensional mapping of the similarity coefficients. Units with a high degree of similarity will be placed close together, units with a minor degree of similarity further apart, and units without any similarity as far as possible from each other. Yet at the same time PCO tries to place all units with no mutual similarity at an evenly spaced inter-point distance. Thus, if the units in the middle have no similarity with the units at either end of the sequence, and the units at either end have no similarity with each other, an arced layout is the only possible way to present the situation. This, of course, is a very simplified explanation of the phenomenon. A stricter mathematical explanation is more complex and difficult to present (Kendall 1971, p. 227).

A CA used on the same ideal data gives a somewhat different organisation of the points on the first two principal axes (Fig. 21.1). They here become organised in something, which is close to a parabola, and probably would have been a parabola, had it not been for the edge effect of the input matrix. Hill (Hill 1974, p. 348) has argued that given an underlying natural gradient in the data, the latter axes in a CA may be approximate polynomials of the first.

The parabola-shaped formation seen in Fig. 21.1 consists of 50 points. However, in reality it represents 100 points, as both units and variables are present. Due to the perfect symmetry of the input matrix, however, the units and the variables cover each other perfectly in the plot. This is of course a situation never encountered with real-world data.

Comparing the two methods, I find that CA is far better than PCO. This is not only due to the symmetric nature of CA, allowing a simultaneous ordering of both units

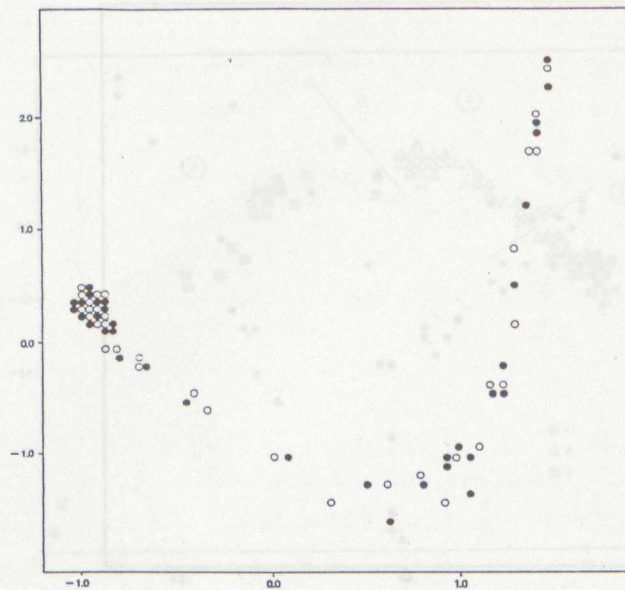


Figure 21.2: Plot of the first two principal axes from a CA of 35 Early Bronze Age hoards (heavy signature) described by 33 types (open signature). After Madsen 1988, Figure 12.

and variables. Rather, the case seems to be that the CA method has a sounder way of data pretreatment than has the PCO. I shall not discuss the specifics of these differences, but demonstrate the difference of results with a practical example. In Fig. 21.2 and Fig. 21.3 the first two principal axes of a CA and a PCO respectively of 35 Early Bronze Age hoards containing 33 metal types are shown (data from Vandkilde 1986). Obviously the CA presents a much better seriation, although of course, we have no means to tell which of the two is the one that presents the truest picture of the data. Theoretically it could be that the less structured representation given by the PCO is the one closest to reality. However, given my experience with the CA I doubt that very much.

By now there seems to be sufficient evidence from the experiments carried out over the last few years at the institute in Århus, to suggest that CA is indeed rather reliable. CA has proved to present excellent results where it has been possible to test these through independent sources. And further it has in some cases convincingly failed, even though an iteratively sorted matrix looked all right. A couple of examples can illustrate these points.

Through his careful studies of the huge Iron Age Slusegård cemetery, Jens-Henrik Bech (1988) has been able to outline a detailed chronology for the C1 period of the Iron Age (appr. 150–250 AD) based on the pottery in the graves. From stylistic studies, manual matrix orderings, and cross-dating with brooches, he created a minute local chronology, dividing the 100 years long period into 4 sub-phases (called 4a, 4b, 5a, and 5b in the local chronological scheme). At this point of the investigation, the data were subjected to a CA, which yielded a very clear seriation (Fig. 21.4) corresponding very well with the results already obtained by Jens-Henrik Bech, but nevertheless suggesting minor improvements and clarifications to the scheme.

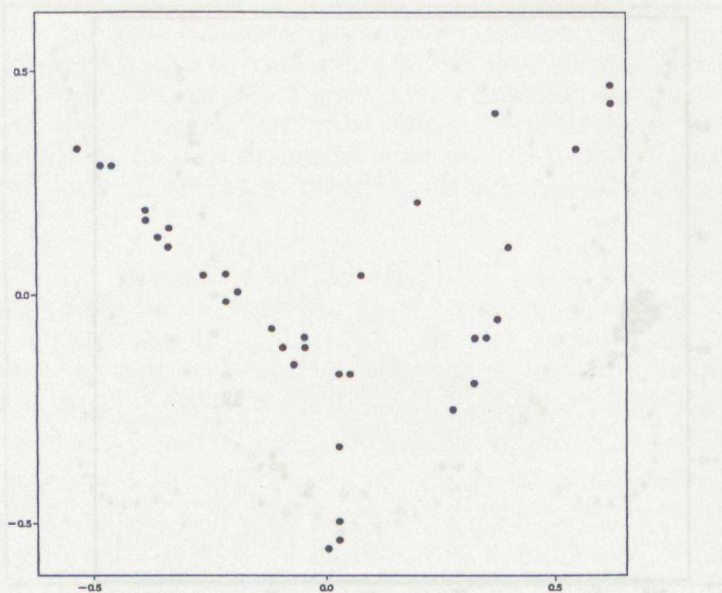


Figure 21.3: Plot of the first two principal axes from a PCO of 35 Early Bronze Age hoards. After Madsen 1988, Figure 13.

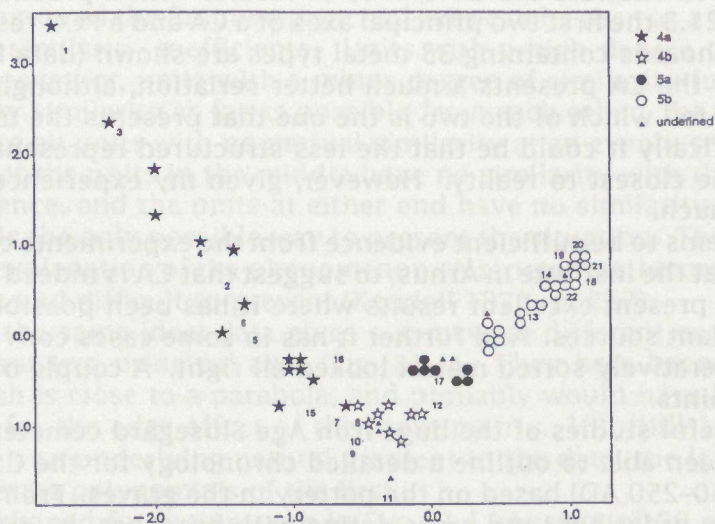


Figure 21.4: Plot of the first two principal axes from a CA of 22 pottery elements in 57 graves from the Slusegård cemetery. After Bech 1988, Figure 2.

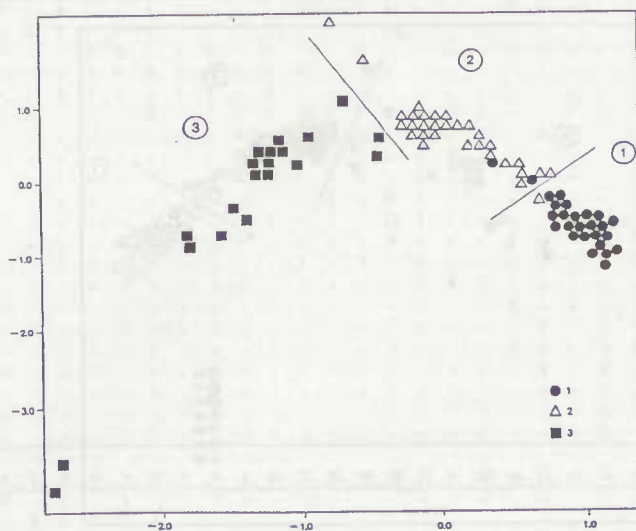


Figure 21.5: Plot of the first two principal axes from a CA of 39 types of ornaments in 77 female graves from the Younger Iron Age with main phase division after Ørsnes 1966 shown. After Nielsen 1988, Figure 10.

Another example is provided by Karen Høiland Nielsen (1988). She set out to check an older chronological study by Mogens Ørsnes (1966), concerned with the Germanic Iron Age. This case has a special interest, since Ørsnes, although he very carefully defined his stylistic elements (ornaments in graves), and presented them in partly ordered matrices, never referred to, nor probably knew anything of formal seriation techniques. He simply based his method on standard Scandinavian approaches. When subjected to a CA the data presented themselves in a nice parabola-shaped layout (Fig. 21.5), and what is more, the main phase division suggested by Ørsnes is clearly substantiated with only minor discrepancies. Clearly the traditional intuitive approach was here sufficient to cope with the chronological ordering of the graves, and the correspondence with the results presented by the CA is astonishingly good. Turning to the subdivision of the main phases given by Ørsnes, however, there is not the same correspondence with the CA results (Fig. 21.6). Clearly the subdivision suggested by Ørsnes cannot be argued along the same lines as the main division.

A final example which shows, that CA may indeed fail where the input data are not sufficiently sound is also presented by Karen Høiland Nielsen (1988). She tried to seriate hoards of golden bracteates from Jutland according to the type of bracteates. The CA did not present a plot that could be reasonably interpreted as a seriation (Fig. 21.7), and one has to conclude that the criteria of continuity do not apply to the hoards of bracteates as currently described by their type inventory. If one looks at the ordered matrix of the input data, however, it is very tempting to assume that a good seriation could be obtained. Only when one has worked with the data for a while is it realized that different orderings of the matrix, each apparently convincing, could be obtained (Table 21.2), and of course none of them represents seriations. The point is that a visually convincing ordering of a matrix is not necessarily a seriation. It takes a method that can fail to prove or disprove this.

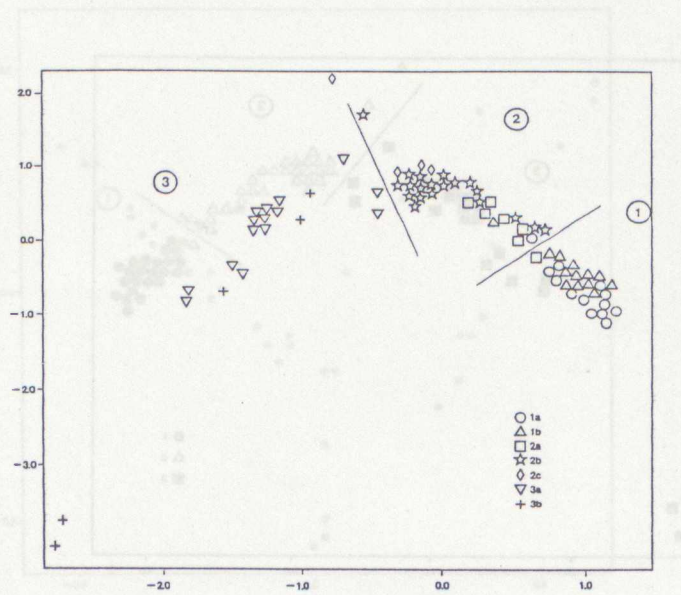


Figure 21.6: Plot of the first two principal axes from a CA of 39 types of ornaments in 77 female graves from the Younger Iron Age with sub phase division after Ørsnes 1966 shown. After Nielsen 1988, Figure 11.

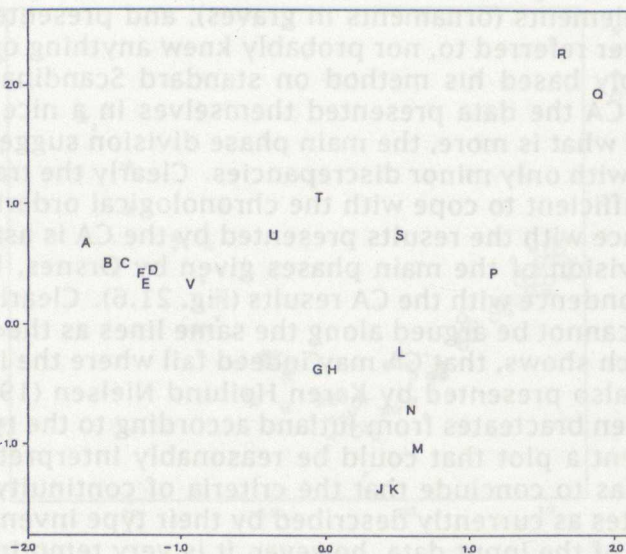


Figure 21.7: Plot of the first two principal axes from a CA of 21 hoards containing 21 different types of golden bracteates. After Nielsen 1988, Figure 12.

	24	23	22	19	18	17	21	20	14	11	15	16	13	12	10	9	6	8	7	25	5	4	3	2	1
U	1	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	-	-	-	-	-	-
T	-	1	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
R	-	-	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Q	-	-	-	1	-	9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
S	-	-	2	1	-	-	1	2	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P	-	-	-	-	2	-	-	-	-	1	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
L	-	-	-	-	-	-	-	1	1	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
I	-	-	-	-	-	-	-	-	5	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-
N	-	-	-	-	-	-	-	-	-	-	2	2	-	-	-	-	-	-	-	-	-	-	-	-	-
M	-	-	-	-	-	-	-	-	-	-	5	-	3	-	-	-	-	-	-	-	-	-	-	-	-
K	-	-	-	-	-	-	-	-	1	-	-	-	12	1	-	-	-	-	-	-	-	-	-	-	-
J	-	-	-	-	-	-	-	-	-	-	-	9	-	-	-	-	-	1	-	-	-	-	-	-	-
H	-	-	-	-	-	-	-	-	1	-	1	-	3	-	-	-	-	-	-	-	1	-	-	-	-
G	-	-	-	-	-	-	-	-	1	1	-	-	3	8	1	-	-	1	-	-	1	-	-	-	-
V	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	1	-	-	-	-	-
E	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	1	-	-
D	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	1	-	3	2	2	1	-
F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	2	-	1	-	-
C	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1	-	-
B	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	1	1	1	8
A	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	2

	1	2	3	4	5	25	7	8	6	24	23	22	19	18	17	21	20	14	11	15	16	13	12	10	9	
A	2	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
B	-	8	1	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
C	-	-	2	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
F	-	-	1	-	2	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
D	-	1	2	2	3	-	1	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
E	-	-	1	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
V	-	-	-	-	-	1	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
U	-	-	-	-	-	-	2	-	1	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
T	-	-	-	-	-	-	-	2	-	1	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
R	-	-	-	-	-	-	-	-	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Q	-	-	-	-	-	-	-	-	-	-	1	-	9	-	-	-	-	-	-	-	-	-	-	-	-	-
S	-	-	-	-	-	-	-	-	-	2	1	-	-	1	2	-	-	2	-	-	-	-	-	-	-	-
P	-	-	-	-	-	-	-	-	-	-	-	2	-	-	-	-	-	-	-	1	1	2	-	-	-	-
L	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	1	6	-	-	-	-	-	-
I	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	-	-	-	2	-
N	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	2	-	-	-	-	-
M	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	3	-	-	-	-
K	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	12	1	-	-
J	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	-	-	-
H	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1	-	-	-	-
G	-	-	-	-	-	-	1	-	1	-	-	-	-	-	-	-	-	-	-	1	1	-	-	3	8	-

	20	21	17	18	19	22	23	24	1	2	3	4	5	25	7	8	6	9	10	12	13	16	15	11	14	
P	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	1	-	-
S	2	1	-	-	1	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-
Q	-	-	2	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
R	-	-	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
T	-	-	-	-	2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
U	-	-	-	-	-	3	-	1	-	-	-	-	-	-	-	-	-	2	-	-	-	-	-	-	-	-
A	-	-	-	-	-	-	-	2	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
B	-	-	-	-	-	-	-	-	8	1	1	1	-	-	-	1	-	-	-	-	-	-	-	-	-	-
C	-	-	-	-	-	-	-	-	-	2	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
F	-	-	-	-	-	-	-	-	-	1	-	2	-	1	-	-	-	-	-	-	-	-	-	-	-	-
D	-	-	-	-	-	-	-	-	1	2	2	3	-	1	4	-	-	-	-	-	-	-	-	-	-	-
E	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
V	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	3	-	-	-	-	-	-	-	-	-	-
G	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	1	8	3	-	-	-	1	1	-	-
H	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	3	-	1	-	-	1	-	-	-
J	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	9	-	-	-	-	-	-
K	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	12	-	-	-	1	-
M	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	5	-	-	-
N	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	2	-	-	-
I	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	-	-	5	-
L	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	1	1

Table 21.2: Matrix of 25 types of golden bracteates in 21 hoards sorted in three different ways. After Nielsen 1988, Table 5.

In conclusion, I should like to emphasize that I find the multivariate techniques to be very powerful methods for seriation purposes. So far as I can see, they are superior to other methods, and especially the CA method gives very fine results. Further, I should like to point out that although the use of formal seriation techniques has been rare in Scandinavia, and although the introduction of multivariate techniques is very recent, the basic principles are not new. Indeed the principles lying behind seriation have been a part of traditional Scandinavian approaches for over a century.

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