PATTERNS ON THE RIM, BASE AND SIDES OF HEMISPHERICAL WOODEN BOWLS

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This paper gives a brief summary of my solution to a problem set by John Barber of the Central Excavation Agency in Falkirk. During excavation he uncovered a number of Mediaeval wooden bowls in various stages of decomposition and deformation. He wanted to visualise the idealised patterns formed by the tree rings on the surface of such bowls, and so he approached me for a computer simulation of bowl carving.

His bowls have a flat circular base, a parallel flat annulus forming a rim and between them the sides of the bowl are part of a sphere (see Fig. 1). The tree is defined to be a series of n co-axial cylinders of radii (in 'inches') r_1, r_2 ,...r_n ; their common axis being the z-axis of three dimensional Cartesian space. Hence the general point on the i'th cylinder is:

$$\{ (r_i \cos \theta ,\ r_i \sin \theta ,\ z) \mid 0 \leqslant \theta < 2\pi,\ -\infty < z < \infty \}$$

The curved side of the bowl is part of a sphere centred at the (vector) point $\underline{c} = (x_c\ ,\ y_c\ ,\ z_c)$. To standardise the scale we assume that the radius of sphere is 4 'inches'. Thus a general point $\underline{x} = (x\ ,\ y\ ,\ z)$ on the sphere is given by:

$$| \underline{x} - \underline{c} | = 4$$

or equivalently:

$$(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = 16.$$

Another point $\underline{p} = (x_p\ ,\ y_p\ ,\ z_p)$ is specified so that the vector $\underline{p} - \underline{c}$ is normal to (i.e. perpendicular to) the planes that form the rim and base. These two planes are identified by the numbers f_1 and f_2, those fractions of the sphere radius, where the rim and base planes cut the vector from $\underline{c}$ to $\underline{p}$. If f_1 and f_2 are negative then the rim and base plans are on the opposite side of $\underline{c}$ to $\underline{p}$. Hence the equations of these planes are:

$$(\underline{p} - \underline{c}) * (\underline{x} - \underline{c} - \frac{4f_i\ (\underline{p} - \underline{c})}{| p - c |}) = 0$$

where * is the scalar product and i is 1 or 2:

$$0 \leqslant f_1 < f_2 \leqslant 1 \quad \text{or} \quad -1 \leqslant f_2 < f_1 \leqslant 0$$

The base is then a disc of radius $4\sqrt{(1 - f_2^2)}$ and the rim is an annulus formed between two circles of radii $4\sqrt{(1 - f_1^2)}$ and $4f_3\sqrt{(1 - f_1^2)}$, for some value of $f_3 \geqslant 1$.

Having thus defined the constituent parts of the bowl, the solution of how to draw the object reduces to solving two separate problems:

(i) To find the loci of the points of intersection of the cylinders and the planes of the rim and base. In general these loci will be ellipses but in the special case when $z_p = z_c$ they will be pairs of straight lines.

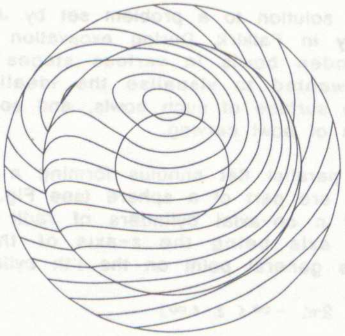
(ii) To calculate the loci of the points of intersection of the cylinders and the sphere. The general point on the i'th cylinder is:

$$(r_i \cos \theta\ ,\ r_i \sin \theta\ ,\ z_c \pm \sqrt{16 - (r_i \cos \theta - x_c)^2 - (r_i \sin \theta - y_c)^2}\ )$$

for some value of θ, where $0 \leqslant \theta \leqslant 2$, provided that the square root has a real value.

Naturally the whole of space has to be transformed so that the observer is looking along the lines from $\underline{c}$ to $\underline{p}$ (Angell 1981). For each cylinder, points are calculated around the outer annulus (the rim), the inner circle (the base) and the inner annulus (the hemispherical side of the bowl) in the transformed space. Care has to be taken with edge effects in each of these three separate cases, and a curve (if one exists) is drawn by joining the points in each of the three cases.

1



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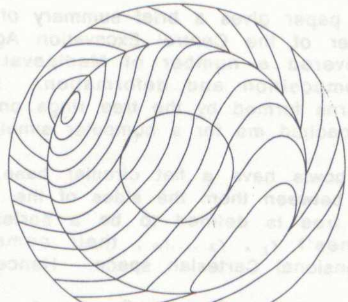
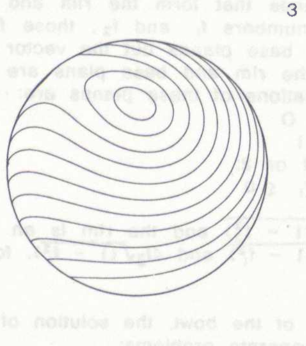


Fig. 1 shows the bowl cut from a tree with rings of radii 0.5, 1.0, 1.5, ..., where $\underline{c}$ = (2, 2, 2), $\underline{p}$ = (1, 2, 0.5), f_1 = 0.1, f_2 = 0.9 and f_3 = 1.3.

If the signs of both f_1 and f_2 are changed (f_1 = −0.1 and f_2 = −0.9) we get Fig. 2. It is as though we cut the tree in half along the equitorial plane of the sphere and then made a bowl out of each part.

3



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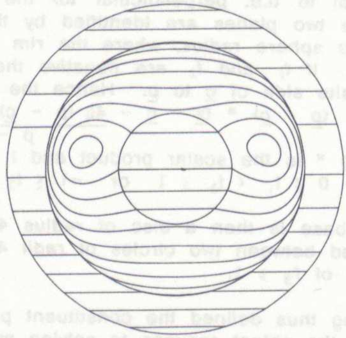
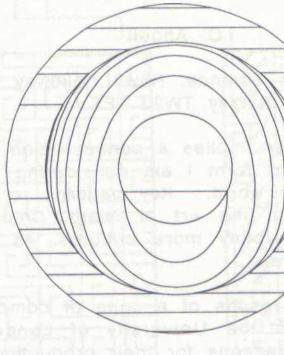


Fig. 3 shows the case when f_1 = 0.0, f_2 = 1.0, f_3 = 1.0 and the values of $\underline{c}$, $\underline{p}$ and the tree ring radii are the same as above, i.e. a hemispherical bowl without a rim or a base.

The special case when $\underline{p} - \underline{c}$ is perpendicular to the z-axis (i.e. $z_p = z_c$) is considered in Figs. 4 and 5. The tree is again defined to have radii 0.5, 1.0, 1.5, etc., but now $\underline{c}$ = (2, 2, 2) and $\underline{p}$ = (1, 0.5, 2) and f_1 = 0.1, f_2 = 0.9, f_3 = 1.3 for Fig. 4 while f_1 = −0.1, f_2 = −0.9, f_3 = −1.3 for Fig. 5.

5



Armed with the program, research workers will be able to understand the topology of the patterns in wooden bowls. I will therefore publish a full listing of the FORTRAN program as soon as possible (Angell 1982), however, anyone with an urgent need for this information may write to me directly.

ANGELL, I.O. 1981 A practical introduction to computer graphics. Macmillan, London.

ANGELL, I.O. 1982 A computer simulation of wood carving. Science & Arch. (forthcoming).