SPATIAL ANALYSIS IN IRREGULAR REGIONS

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1. Introduction.

Standard methods for the statistical analysis of spatial data, in particular techniques based upon nearest neighbour distances, usually require unrealistic assumptions on the nature of the area studied. Methods used often require that the area is isotropic and infinite in extent. Edge effects are usually neglected, except perhaps in very simple cases, for example in square or circular regions (Ripley, 1976). Many of these methods were developed initially in the field of ecology. In applications within this area, for example the study of the distribution of species in a large region, such approximations to reality are perhaps justified. However, in archaeological applications the shape of the area studied is typically intrinsically irregular and finite and it would be grossly misleading to pretend otherwise.

The motivation for the methods described here is the statistical analysis of the spatial distribution of over three thousand artifacts and bone fragments which were recorded *in situ* during excavations at Cnoc Coig in Oronsay, a small island in the Inner Hebrides. The excavated and recorded area at Cnoc Coig is very far from being square or infinite in extent; see figures 1 & 2. It is composed of three separated regions containing a total of four blank areas or "holes". Additionally, because Cnoc Coig is not the result of a single occupational episode, differences in the depth distribution of objects within the midden must be considered. Thus a further requirement of the analysis was that the methods should extend to the analysis of the three dimensional spatial distributions of the finds. For reasons outlined below, we concentrate on the inter-relationships between the distributions of two types of artifact, rather than the distribution of any particular single type. Indeed, in many cases questions of whether the distribution of any particular type of bone fragment or artifact was randomly spread over the area, or was more systematic in nature, could be readily answered by visual examination of plots, see for example figure 1. Such plots are of major importance in analysing spatial data and their production can be easily included in the data management system.

2. Archaeological Background.

The Mesolithic shell midden site at Cnoc Coig is one of five such on Oronsay, all of which have been excavated as part of the Oronsay Archaeological Project (see Jardine 1977, Mellars 1978,1981,1983, Mellars & Payne 1971, Mellars, Wilkinson &

Fieller 1980, Peacock 1978). All of these sites are close to the former shore line at the time of the maximum postglacial marine transgression (Jardine 1977). Six radiocarbon dates have now been obtained on charcoal samples from Cnoc Coig and all of these fall within a narrow range from around 3,700 to 3,500 bc (Mellars 1978: 376, 1983).

All of these shell middens have been sampled to obtain palaeoenvironmental data, material for radiocarbon dating and evidence relating to economy and seasonality. In addition, during the 1975 field season, Cnoc Coig was subjected to a more extensive and statistically controlled programme of probabilistic sampling (Peacock 1978). Furthermore, large-scale area excavation was carried out on this site during four field seasons from 1973 to 1979. As a result of the recent intensive investigations at Cnoc Coig, approximately 70% of the area has now been dug.

The three thousand plus items found in situ offer a valuable opportunity to investigate one of these shell middens from the perspective of spatial archaeology. One of the most intriguing and major questions of the "Obanian" sites in general concerns how they may be interpreted in terms of late Mesolithic hunter-gatherer subsistence-settlement systems in Western Scotland. Hence, a main objective of the spatial analysis of Cnoc Coig is the elucidation of the nature of the occupations represented by these shell middens. Within this general area, more specific questions arise concerning the overall distribution of various categories of items in the midden. In particular, can distributions be attributed to the use of different modes of disposal? Are certain types of artifacts associated and do particular artifacts tend to occur in specific localised areas of the midden? These and many other questions concerning the activity structure and the organisation of space of the occupations represented at Cnoc Coig can be asked of the data recorded in situ from area excavations at the site.

The particular problem we study here is the relationship between two types of limpet scoop. A fundamental question is why more than one type of material was used for these artifacts. One possible explanation is that one type of material (antler) was preferred but was in limited supply, while the other type (stone, which was locally available) was used when supplies of the preferred type were temporarily exhausted. The implication of this simple model for the spatial distribution of the different types of scoop is that there should be observable segregation between the two types.

3. Statistical Analysis.

3.1 Initial Considerations

Some of the questions posed above can be answered adequately, and indeed most easily, by simple visual inspection of plots. For example, Figure 1 shews the distribution of bone fragments from three species of bird. It is obvious from the plot that the species are highly localised in the midden and do not associate or segregate in any meaningful fashion. It would be a futile waste of time to perform any sophisticated statistical analysis on these data. Figure 2, by contrast, shews the distribution of antler and stone limpet scoops in one particular level from 70.0 to 79.9cm below datum. It is not apparent to the eye whether the presence of one type "inhibits" the presence of the other or whether they are intermingled. Such segregation or attraction may be present in some parts of the site and not in others. It is not easy to see the nature of the relationship between the distributions of the two types of scoop and so a more detailed analysis is required to aid the common sense interpretation of such plots.

We concentrate here on the analysis of spatial patterns consisting of two types of points. The reasons for this are firstly that the important archaeological problem of the relationship between the two types of limpet scoop demands this. Secondly, simple analysis of randomness or otherwise of single type point patterns could, in our case at least, usually be easily performed informally and graphically. Finally, it is admittedly perhaps the easiest problem to tackle in the analysis of patterns in highly irregular regions. This is because the assessment of statistical significance of measures of pattern features has to be performed by simulation, and with two types of points a random relabelling method lessens the computational effort considerably. However, many of the ideas and procedures for correcting for edge effects would apply equally well in the analysis of single type point patterns.

The form of our analysis of the pattern of distribution of the two types of limpet scoop is the following. Step one is to calculate some numerical measure of the degree of segregation in the pattern of the actual scoops found. Step two is to generate an artificial pattern of points whose distribution matches the observed one in all but one respect. The exception is that the artificial pattern should be non-segregated, or rather that the segregation in the artificial pattern should be attributable only to random variation. The same numerical measure of segregation is calculated for this random pattern and recorded. Step two is repeated a large number of times, in our study 499 times, and the complete set of 500 numerical measures is sorted into rank order. If the value from the actual pattern is larger than say 95% of those obtained from random patterns, then we conclude that the actual pattern exhibits a "significant" degree of segregation, in fact significant at the 5% level. This form of statistical test, known as a "Monte Carlo test", provides a pragmatic approach when the theoretical distributions involved are too intractable for exact tests to be calculated. They are widely used in spatial analysis of all forms, see for example Besag & Diggle (1977).

The details of the analysis that have to be determined are firstly which measure of segregation to use and secondly how to generate "look-alike" but non-segregated random patterns. A separate issue is how to ensure that the measure of segregation used is not too dependent on the shape of the region studied, i.e. how to apply an "edge correction". This last feature would appear to be of particular importance in the Cnoc Coig study because the convoluted nature of the boundary means that a large proportion of the study area is very close to a boundary.

3.2 Segregation Measures

A variety of segregation measures are available. These

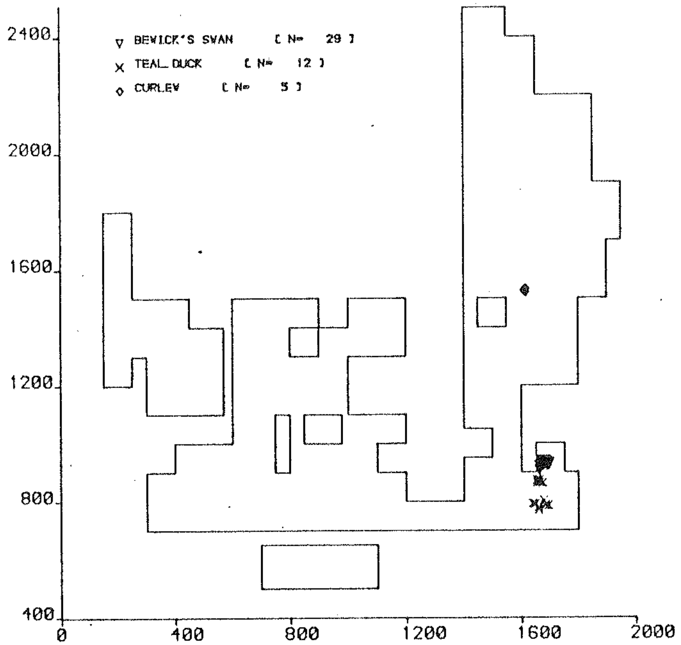

Figure 1: Cnoc Coig Excavated Area

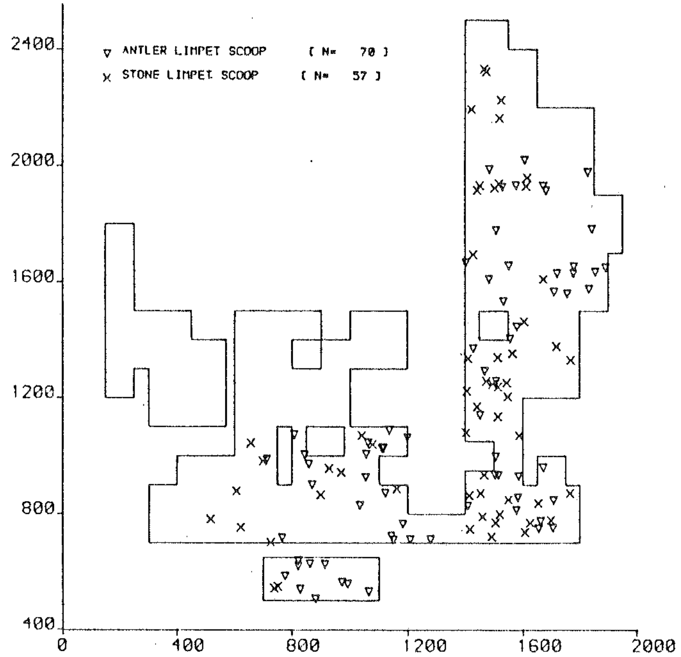

Figure2: Limpet Scoops

are generally of one of two types; either they are based on quadrat counts or they are based on inter-point distances, perhaps on just the first or second nearest neighbour distances. Simple such measures are described in Pielou (1969) and Hodder & Orton (1976). Ripley (1981) discusses the relative merits of quadrat and distance based methods. In this study we considered only distance based methods because they are more easily generalised to three dimensions and because of the difficulty of defining a system of quadrats that were entirely interior to the study area. The particular measure we use is Pielou's coefficient of segregation, (Pielou 1969: 182) based on both first and second nearest neighbours, although our methods would extend equally well to the Ripley K-statistics and empty space methods described by Lotwick & Silverman (1982).

First we describe the basic form of the coefficient and then consider the modifications required to allow for edge corrections. Each point of the pattern is examined and the type of its nearest neighbour is determined. The coefficient is based upon the number of "mixed pairs", i.e. where one point has a different type of point as its nearest neighbour. It is defined as

$$S = 1 - \frac{\text{observed number of mixed pairs}}{\text{expected number of mixed pairs}},$$

the expected number being calculated on the assumption of random mixing. If we display the various forms of nearest neighbour pairs in a two-way table:

		type of nearest neighbour		
		A	B	total
Type of base point	A	a	b	m
	B	c	d	n
	total	r	s	N

then we have that

$$S = 1 - \frac{N(b+c)}{(ms+nr)}$$

It is easy to see that S ranges in value from +1 when every point has the same type as its nearest neighbour (b=c=0), to -1 when the whole population is composed of isolated mixed pairs. To assess whether a given value of S for an actual pattern represents evidence of segregation or whether its value is no greater than might have arisen by chance, we apply the Monte Carlo or simulation test described above and whose details are given in 3.4 below.

In passing we note that the table above is NOT a contingency table and it is incorrect and misleading to apply a χ^2 test of independence to it, notwithstanding the suggestion to do so made by Pielou (1969: 182) and repeated by Hodder & Orton (1976: 204). The reason is that if point j is the nearest neighbour of point i, then it is more than probable that point i is the nearest neighbour of point j (unless there is a third point close to j but on the 'other side' of it from i). Consequently, most nearest neighbour distances are in effect counted twice in the table above, thus inflating the χ^2 statistic by a factor of nearly 2 and therefore producing spurious evidence of "segregation". Similar comments apply to similar tables based on quadrat counts, and generally the complete set of nearest neighbour distances cannot be regarded as independent observations so care is needed in their analysis.

It is obvious that the coefficient of segregation S applies equally well to three dimensional data. Furthermore, a corresponding coefficient, S_2 say, can be calculated in terms of second nearest neighbours. This measures segregation on a larger scale and is independent of the small scale micro pattern of the distribution. Of course the coefficient could be calculated in terms of any order of nearest neighbour and sequential or stepwise use of these would detect segregation of 'clusters' of the corresponding size.

Our analysis was performed in terms of the two statistics S and S_2 . We looked at both the complete three dimensional distribution of scoops throughout the midden as well as in various subregions and at the (essentially) two dimensional distributions in separate levels of 10cm.

3.3 Edge Corrections and Winding Numbers

The practical validity of the measures of spatial segregation given above rely upon knowing without uncertainty the type of the nearest neighbour for any given base point. If the base point is closer to a boundary than its distance from its nearest neighbour within the excavated region, then there is always the possibility that there might be an artifact, as yet unexcavated, which is closer to the base point than that currently identified as its nearest neighbour. The edge correction method we propose is designed to reflect this uncertainty for those points which are very close to the boundary. There is a slight difference between the two and three dimensional cases.

If a base point is closer to a boundary than its distance from its nearest neighbour within the excavated region then it is plausible to permit that particular nearest neighbour distance to contribute less weight to the measure of spatial pattern, than if its nearest neighbour were known with certainty. We suggest that the appropriate weight to use is the proportion of the area of the disc, (or volume of the sphere in three dimensions) which is centred on the base point and has radius equal to the apparent nearest neighbour distance, which is contained as interior to the excavated region. Points which are closer to their neighbour than to a boundary would thus have weight 1, otherwise the weight is less than 1. Ripley (1976, 1981) suggests that the appropriate weight should be the proportion of the circumference contained as interior. The difference is slight, except that the former is easier to calculate.

To calculate the proportion of the area of a disc of given centre and radius which lies within the study region is not difficult to do algebraically if the region has a simple convex shape, circular or rectangular for example. This is not the case with the Cnoc Coig site and the analytical difficulties of providing exact calculations would be formidable. We propose an approximate method based on the calculation of 'winding numbers'.

The winding number of a point with respect to a closed curve is the number of times that the curve passes completely around the point. So, if the point lies outside the curve its winding number

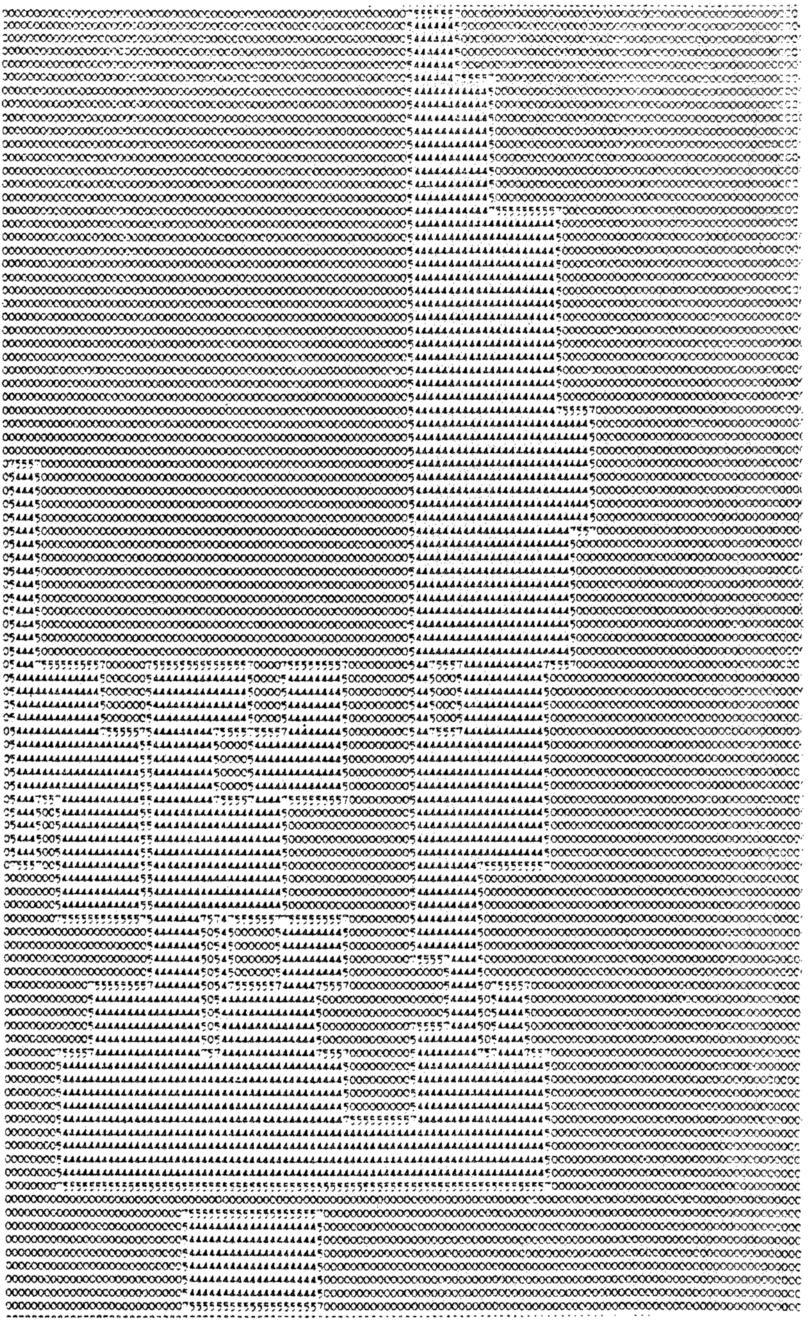

Figure 3: Winding Numbers for Cnoc Coig Excavated Area

is 0, if it lies inside the curve the winding number is greater than 0. Thus, the winding number of a point indicates whether the point is interior or exterior to the region enclosed by the curve. The application of winding numbers to the current problem is as follows. Suppose the complete area is divided into a large number of small cells, for each cell calculate the sum of the winding numbers with respect to each of the closed curves which define the study region, boundaries of regions are taken anti-clockwise, and boundaries of holes within regions are taken clockwise. Then for any given disc in the region, the cells making up the disc can be determined, their winding numbers examined and thus the proportion of cells of that disc which are interior to the excavated region can be calculated, thus giving the appropriate weighting for that nearest neighbour distance. By making the cells sufficiently small, the proportion of the area of the disc inside the region can be approximated to any required accuracy. The procedure in three dimensions is essentially identical, regarding the sphere as composed of levels of cells.

For the Cnoc Coig site we calculated the winding numbers as follows. We divided the site by a 250 by 250 grid into 62500 cells, so that each cell corresponds to a square on the ground of about 8cm. This is sufficiently small since the majority of nearest neighbour distances are more than 20cm. These 'cells' were represented in the computer by a 250 by 250 array which ultimately contains the winding numbers for every 8cm square on the site. We took each cell corresponding to a point on the boundary in turn, travelling anti-clockwise for boundaries of regions, clockwise for boundaries of holes. At each cell of the boundary the following calculations were made; 1 was added to each cell to the left of the boundary point and 1 was subtracted from each cell to the right of the boundary point, left and right being measured from the direction of travel. The result is that when all boundaries have been travelled over, all cells interior to the region have a score (or essentially a winding number) of 4, those outside the region or within the holes in the region have a score of 0. Cells corresponding to boundary points have to be treated separately; we arbitrarily designated corner cells by the digit 7 and other boundary cells by the digit 5. Figure 3 shews the resulting digitised map of the Cnoc Coig site (although for the purposes of illustration this is calculated from a 100 by 120 grid).

3.4 Monte Carlo Simulations by Random Relabelling

To assess the significance of the coefficient of segregation **S** calculated from m points of type A and n points of type B, the Monte Carlo procedure requires an artificial set of points with the same number of each type and placed in the same region subject to the same boundary constraints and weightings as the original. One method would be to place m+n points randomly in the region, the first m being designated type A. To ensure that a randomly placed point was actually in the excavated region and not in one of the holes or outside it altogether the winding number for the corresponding cell could be checked. This procedure would be computationally expensive, particularly since the nearest neighbours and the weights for each distance would have to be determined afresh for each of the 499 simulated patterns.

An alternative procedure which is sufficiently fast to be run interactively at the terminal, and thus be included as part of the data management system, is the random relabelling procedure. This takes the given positions of the points, and then each simulation consists of selecting randomly m points from the set of m+n and labelling them as type A, the others being regarded as type B, the coefficient S is calculated from this randomly relabelled set. This restricted simulation method is slightly different in philosophy from that above, and produces a test of segregation conditional upon the observed overall pattern of finds. The procedure is very fast because which points form nearest neighbour pairs and the appropriate weightings are not affected by the relabelling and so need be calculated once only.

3.5 Results

The detailed results of the segregation study in terms of all the separate levels and the various subregions of the site will be presented elsewhere with a proper assessment of their archaeological implications. The overall conclusion was that certain parts and levels of the site exhibited some strong evidence of segregation between the two types of scoop. For illustration of the effect of weighting we present here the results from just one level and the results from the complete three dimensional distribution, both with and without the correction for weightings, and for both nearest and second nearest neighbour measures. A general feature was that the edge corrections altered the significance of the results little; the greatest differences were observed on the calculations based on second nearest neighbour differences.

level 70.0-79.9cm B.D.: 127 scoops

		unweighted					weighted			
		nearest point		2nd nearest			nearest point		2nd nearest	
b		ant.	st.	ant.	st.	tot	antler	stone	antler	stone
a	antler	48	22	45	25	70	42.3	21.2	36.9	23.9
s	stone	24	33	25	32	57	21.8	31.3	22.5	28.3
e	total	72	55	70	57	127				
		$S=0.2655$		$S_2=0.2043$			$S=0.2554$		$S_2=0.1645$	
Sig. level=.014				.010			.014		.032	

three dimensional distribution: all 589 scoops.

		unweighted					weighted			
		nearest point		2nd nearest			nearest point		2nd nearest	
b		ant.	st.	ant.	st.	tot	antler	stone	antler	stone
a	antler	165	128	155	138	293	159.0	121.1	143.6	130.1
s	stone	132	164	136	160	296	127.1	156.6	130.1	148.0
e	total	297	292	291	298	589				
		$S=0.1172$		$S_2=0.0696$			$S=0.1197$		$S_2=0.0569$	
Sig. level=.006				.080			.008		.124	

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