On archaeological applications of the Voronoi Tessellation

Ian O. Angell and Richard E.M. Moore.

Royal Holloway College

University of London

Egham, Surrey. U.K.

Voronoi (1908) tessellation of the plane (e.g. figure 1), also known as Dirichlet (1850) or Thiessen (1911) tessellation, is a technique often used (and abused) by archaeologists. The idea is that 'centres' (e.g. sites of manufacturing) scattered over the land, may be regarded (in the absence of known boundaries over the countryside) as holding sway over all parts of the country that lie closer to each particular centre than to any other centre (see figure 2).

By this definition the boundary of influence between two neighbouring centres must be a line which is the perpendicular bisector of the line joining the two centres. As more centres are involved then the problem becomes more complex, and the resultant areas of influence prove to be convex polygonal areas. We stress the term convex, since a number of archaeologists have produced so-called Voronoi tessellations which include concavities (Danks 1977, Hammond 1972, Renfrew 1973). There is no time here to give a complete survey of such applications, instead we will consider algorithms for calculating and drawing such tessellations.

The simplest method, which is eminently suitable for a small number of centres (usually the case for archaeology) is to take any centre and place a large polygon (initially a rectangle) around it. Then arrange the other centres in increasing order of distance from the chosen centre. Taking these ordered centres one at a time, we calculate the perpendicular bisector of the line from this point to the original centre, and use the bisector to slice off part of the surrounding polygon. Eventually the radius of the polygon (the maximum distance from the centre to any point in the polygon) is smaller than the distance of each of the remaining centres from the original centre, and we have the Voronoi polygon for the chosen centre. Chosing each centre in turn will furnish us with the Voronoi tessellation. We demonstrate a BASIC program using this method for drawing a tessellation in Mode 0 on the B.B.C. microcomputer. Copies of this program are available from the authors.

As the number of centres becomes larger then this method proves inefficient. The time taken to put the set of centres into order (for every chosen centre) proves wasteful, since most centres from the ordered set will not be used anyway as they will be at a distance far larger than the final radius of the polygon for the chosen centre. One way around this problem is to divide the centres up into neighbourhoods and store each neighbouhood as a linked list. We can then start in the neighbourhood containing the chosen centre, and move slowly outwards in a manner similar to the first method.

As the number of centres gets larger still, the order of thousands, then this method also tends to be inefficient. The next method, pioneered by Green and Sibson (1978), is to define the whole tessellation as a complex data structure. We interpret the tessellation as two arrays which hold the two-dimensional coordinates of the centres and the vertices of the tessellating polygons together with an array of nodes with one node for each centre. Each node points to two linear lists : the first list is circular and holds the index of the vertices of the polygon surrounding the centre, the second holds a list of indices of centres that are neighbours of the particular centre. To aid our explanation we refer to figures 3a,b in which we use bold numbers to refer to centres and non-bold numbers to refer to polygon vertices. Starting with one centre enclosed in a polygonal boundary, we define the initial structure to be a node pointing to two lists, the first (circular) containing 1,2,3,4 and the second empty. We then successively add the other centres one at a time. With each addition the computer must find the polygon of the present tessellation which will contain the new centre, and this polygon will obviously be split by the perpendicular bisector of the new centre and the centre of the polygon. Neighbours, which can be found directly from the data structure defining the tessellation, will also be altered with corresponding perpendicular bisectors.

We find the neighbours and construct the polygon for the new centre and add it to the array structure, and also adjust the lists in the structure which correspond to polygons in the old tessellation which are altered. See figure 3a which shows the addition of centre 2, and figure 3b which shows the addition of new centre 3. Note in this latter operation vertex 6 is no longer needed - the program must include a garbage collector so that discarded locations in the array of vertices can be re-used, thus minimising waste of store. Output from such a program is shown in figure 1, which shows a tessellation of a plane containing 1000 points.

We are happy to help any archaeologists who need such output. If they write to us at the above address, enclosing planar coordinate data we will be pleased to supply them with a microfilm plot, similar to that used to print figures 1 and 2.

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Fig. 1   SEED=0.123456   NUMBER OF POINTS=1000

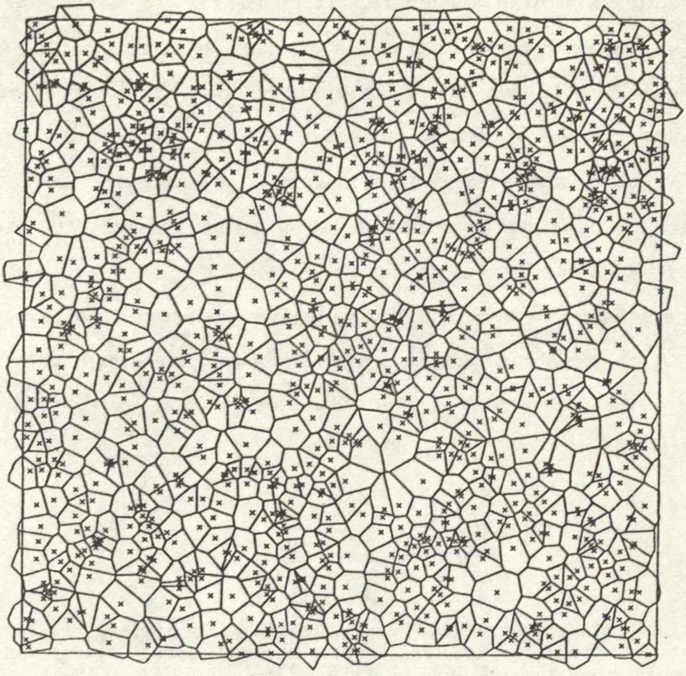

Fig. 2

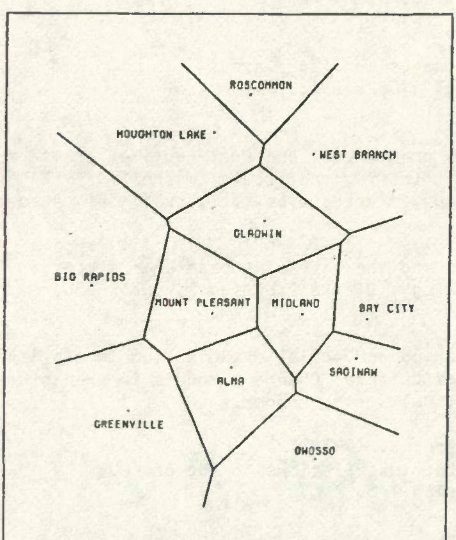

Fig. 3a

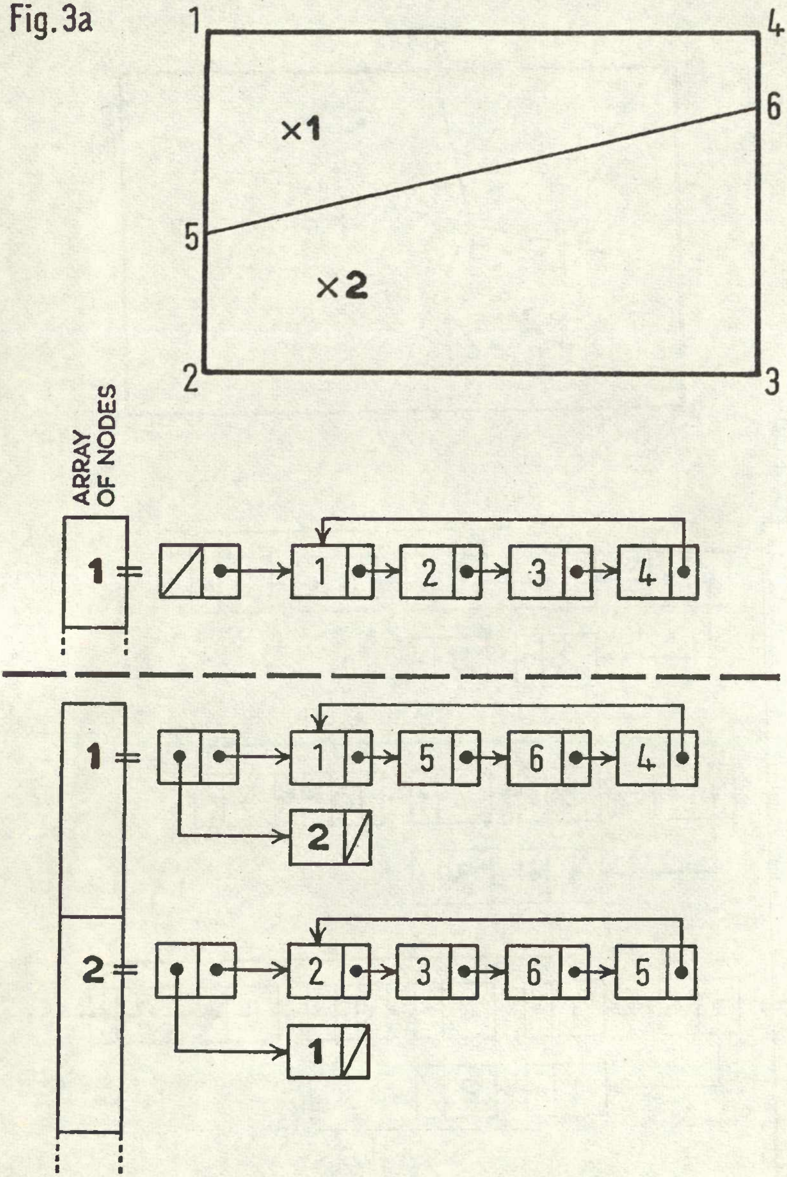

Fig.3b