20

An approach to quantifying window glass

M. J. Baxter

(Dept. of Mathematics, Statistics, and Operational Research, Nottingham Polytechnic, Clifton Campus, Nottingham)

H. E. M. Cool

(Romano-British Glass Project, School of Continuing Education, University of Leeds)

20.1 Introduction

Fragments of window glass are common finds on Romano-British sites, but any contributions they could make to wider interpretations are generally ignored. One of the reasons for this is that the amount found is rarely quantified in a useful way so it is not possible to make inter-site and inter-building comparisons. This paper reports on the development of a method of quantification which may be rapidly applied to assemblages to facilitate such comparisons.

It is clear from the finds of window glass in early securely stratified contexts, for example at Verulamium (Charlesworth 1972, p. 213, no. 14) and Exeter (Charlesworth 1979, p. 229), that buildings with glazed windows were being erected at an early stage in the Roman occupation of Britain. During the first to third centuries the commonest type of window glass was cast. The manufacture of this is relatively simple, unlike that of the blown variety. It was produced by pouring molten glass into a flat frame resulting in a pane that has a matt under surface and a glossy upper surface. The central part of such a pane is often very regular in thickness but towards the rounded edges it often becomes thicker and more irregular, and frequently shows tooling marks on the upper surface where the viscous glass was tooled towards the edges of the frame (Boon 1966).

Cast window glass is translucent rather than transparent, and the majority is blue/green though attempts were sometimes made to decolourise it (Cole 1966, p. 46). The original size of the panes is not known, but one measuring not less than 60 by 60 cm. has been recorded from the Flavian bath-house at Corbridge (Charlesworth 1959, p. 166) and a complete pane from a second century bath-house at Garden Hill, Hartfield, Sussex (Harden 1974, p. 280) may originally have come from a sheet of similar dimensions. It measures 25.5 by 23.5 cm. but has been cut from the corner of a larger pane.

The raw material for these window panes was probably cullet, i.e. broken fragments from vessels and windows. This would have been easily available in Roman Britain. It is known from literary sources that such fragments were collected for re-use in Rome (Price 1977, p. 70), and the archaeological record suggests the same was true in Britain. At Mancetter, Warwickshire, for example, where one of the very few glass furnaces from the western Empire has been excavated, a large quantity of broken vessel glass was found and it appeared that the raw material for the industry had been cullet. The combination of the availability of raw materials and the ease with which cast window glass was produced suggests that it could have been widely available. It would, therefore, be interesting to know which varieties of buildings were regularly glazed as this could contribute to an appreciation of the degree to which the trappings of Roman civilization were adopted in Britain.

It is widely accepted that bath-houses were glazed, as glass in the windows would have served the double purpose of illumination (see Zienkiewicz 1986, p. 122 & footnote 32) and heat conservation. This is reflected archaeologically as the window glass assemblages from bath-houses tend to be larger than those found on other sites in respect of both quantity and size of fragments. The regular occurrence of window glass fragments elsewhere, however, indicates that glazed windows were used in other types of buildings as well. From empirical observation it is clear that window glass is not found uniformly on these other types of site. Though this may in part be due to local depositional history, it is as likely to reflect differences in the original glazing, just as the large quantities of glass regularly associated with bath-houses do. It was therefore thought to be useful to compare the amounts of glass from different sites and, if possible, different types of buildings.

The only way in which window glass has been quantified hitherto is by weight (Harden & Price 1971, p. 367; Zienkiewicz 1986, p. 337). On its own this is not a useful measure as it is not related directly to function, which in this case is area covered, nor is it necessarily comparable from site to site. As noted above the thickness of an individual pane often varies between the centre and edge, and the average thickness of different panes also varies. Given these variations two fragments of the same area may well have significantly different weights.

It was therefore felt that the method of quantification used for the inter-site comparisons must relate to function, and this meant that the area of the fragments had to be used. This raised problems of resources as fragments of window glass normally have very irregular outlines. It is easy to measure the area by placing a fragment on a piece of graph paper, drawing around it and then calculating the area enclosed with the aid of the divisions on the paper. This is, however, immensely time consuming taking on average 5 minutes for a small fragment and 15 minutes for a large one. The amount of time taken to quantify complete assemblages in this way, and the corresponding cost in scarce post excavation resources, could not be justified. The time taken to record the weight and thickness of a fragment, however, is negligible as it can rapidly be measured and recorded during the preparation of the archive catalogue which requires each fragment to be examined. It was thus clear that if a satisfactory and cost effective method of quantifying the area represented was to be developed it would have to be based on the measurement of weight and thickness.

A simple model for predicting area from weight and thickness is first developed, followed by a section on statistical considerations. It is assumed that sampling is necessary and that pieces must be handled individually. While we are aware of measuring techniques that can directly determine area the present paper assumes that these are not readily available to the analyst. The methodology is applied, in various ways, to a bath-house and to a domestic assemblage. The concluding section discusses the issues that need to be addressed when applying the methodology to window glass assemblages.

20.2 The model

If specimens are of known weight, W, uniform thickness, Z, and uniform density then, if A is the area, $W = cAZ$ where c is a constant. This gives rise to the model for area of the form

$$A = \beta x \qquad (20.1)$$

where $x = W/Z$ and β is a constant.

In practice this will not be satisfied exactly because of measurement error *etc.* and regression through the origin will not be assumed so use

$$A_i = \alpha + \beta x_i + \epsilon_i \qquad (20.2)$$

The error term may be more variable for larger pieces and the error variance is assumed to have the form

$$V(\epsilon_i) = \sigma^2 x_i^\gamma \qquad (20.3)$$

In the paper unweighted least squares ($\gamma = 0$) and weighted least squares with $\gamma = 1$ are used, along with an approach for which γ is estimated.

20.3 Theory

Let n specimens, the observed sample, be selected from a total of N. These have A determined, as well as W and Z. The remaining $(N - n)$ specimens, the prediction sample, have only W and Z measured.

Estimates of α and β are obtained from equations 20.1 and 20.2 using standard (weighted) least squares methods. The MINITAB and GLIM packages have been used here but any package with a weighted least squares facility will do.

The predicted total area is then

$$\hat{T} = Ts + \hat{T}p = \sum_s A_i + (N - n)\hat{\alpha} + S\hat{\beta} \qquad (20.4)$$

where

$$S = \sum_p x_i \qquad (20.5)$$

and $\hat{T}$, Ts and $\hat{T}p$ are the total area, and areas of the observed and prediction samples; indicates an estimated quantity; and the s and p indicate whether summation is over the observed or prediction sample.

The variance of the predicted total is

$$V(\hat{T}) = (N-n)^2 V(\hat{\alpha}) + S^2 V(\hat{\beta}) + 2(N-n)SC(\hat{\alpha}, \hat{\beta}) + \hat{\sigma}^2 \sum_p x_i^\gamma \qquad (20.6)$$

where $V(\hat{\alpha})$, $V(\hat{\beta})$ and $C(\hat{\alpha}, \hat{\beta})$ are the variances and co-variances of the estimates. Any package with regression facilities should make $V(\hat{\alpha})$ and $V(\hat{\beta})$ available (as the squares of the standard errors of the estimates) and it is assumed here that, as in MINITAB or GLIM, $C(\hat{\alpha}, \hat{\beta})$ is also available. If $C(\hat{\alpha}, \hat{\beta})$ is not available it is possible to write 20.6 in terms of $V(\hat{\alpha})$ and $V(\hat{\beta})$ only, at the expense of complicating the expression.

In any event 20.5 and 20.6 are easily calculated after a regression fit. For special cases 20.6 simplifies somewhat; in particular for regression through the origin we get

$$V(\hat{T}) = S^2 V(\hat{\beta}) + \hat{\sigma}^2 \sum_p x_i^\gamma \qquad (20.7)$$

The theory here is essentially that given in Royall (1970). It is possible to extend the analysis further by estimating γ using the theory and macros given in Aitkin (1970) for the GLIM package, and this is done in some of the later examples.

20.4 Practical considerations

The aim is to devise a relatively simple method for estimating the total area of a window glass assemblage when there are a large number of pieces. The method should require relatively few area measurements; should be easily and quickly applied; and allow for an estimate of the standard error of the total.

Questions that arise include how many pieces to sample; which pieces; whether or not to use regression through the origin; and what form of weighting to use. To help answer such questions experimental work was undertaken on glass from two sites. This work is described in the next three sections.

20.5 Experiments with population data

To obtain an idea of the time saved by sampling, and to provide material for some sampling experiments, an assemblage of window glass from the Roman bath-house at Catterick was exhaustively analysed. The 151 specimens used were weighed to the nearest 5g. Thickness was taken as the average of the minimum and maximum thicknesses.

The total area was 3320 cm^2. In later discussion it may help to think of this kind of measure in two ways; either as a (hypothetical) 58 by 58 cm. pane, or as about three and two-third 'notional panels' of 30 by 30 cm. This latter concept is tentatively identified with the size of panel used in making a window.

About 80% of the pieces weighed 25g or less, with 10% 75g or more. The skewed nature of the distribution is typical. The proportion of large pieces and length of the tail is likely to depend on the type of assemblage, with assemblages of domestic glass having the majority of pieces

30g or less in weight. The 25–30g weights correspond roughly to a W/Z ratio of 6.

The correlation between area and W/Z was 0.993 and the plot of the two clearly linear. Any sensible method of sample selection, in this case, is likely to lead to a reasonable prediction of T. A high correlation is not guaranteed, however, and it is worth investigating which methods of sample selection are to be preferred. For what follows the data are assumed to arise from a 'superpopulation' model described by equation 20.2.

Fitting unweighted regressions to the data and subsets of it, followed by plots using standard regression diagnostics (e.g. Atkinson 1985), established clearly that some form of weighting was desirable. To investigate this the model defined by 20.2–20.3 was fitted in GLIM using the macros given in Aitkin (1987). The full data set and subsets defined by whether or not W/Z was less than 6 were used, and models with and without a constant were fitted. Results are reported in table 20.1.

The results for the separate subsets are clearly different and do not support the use of regression through the origin as both constant terms differ significantly from zero at the 1% level. The value of $\hat{\gamma}$ is consistent with a 'true' value of 1 in both cases. Using all the data regression through the origin looks appropriate but the estimate of γ significantly exceeds 1, presumably because of the effect of the larger pieces. Results obtained assuming $\gamma = 1$ (not shown) were broadly similar to those in Table 1 and, in the next section, this simplifying assumption will be made.

The results suggest that especial attention may need to be paid to the larger pieces. They are likely to be influential in any analysis and their behaviour may differ from the bulk of the smaller pieces, so some form of separate treatment may be needed.

In the final section the conclusions we draw from these observations and the other analyses are summarised.

20.6 Sampling experiments

Royall (1970) gives some theory concerning the optimal choice of sample to minimise $V(\hat{T})$. This presumes that the true model and weights are being used and we will wish to examine rather than assume this. A range of sampling strategies were thus examined using the Catterick data. These were:

1. Select the largest 20 specimens as measured by W/Z.
2. Select the 10 smallest and 10 largest specimens.
3. Select every 8th or 9th specimen, ordered on W/Z.
4. Measure, but do not use in the regression, the 10 largest pieces. Select the 5 smallest and 5 largest pieces from the remainder.

Strategy 1 is Royall's optimal strategy for regression through the origin; 2 approximates to the kind of strategy sometimes arising in optimal design for regression; 4 is similar to 2 with stratification with the most important pieces being measured exactly and not influencing the regression; 3 allows checks on the integrity of the regression not possible with other procedures. Other strategies, involving stratification and separate estimation within each stratum, are possible.

Results for weighted regression assuming $\gamma = 1$ are given in table 20.2. The difference between the smallest and largest estimates of nearly 670 corresponds to a pane of about 26 by 26 cm. The worst single estimate differs from the true value by approximately a 20 by 20 cm. pane. In the context of an actual area representing about three and a half notional 30 by 30 cm. panels these differences are not negligible.

The results for strategy 1 with a constant term in the regression illustrate the dangers that may arise from using the strategy. Essentially the problem of an imprecise and inaccurate estimate arises because the estimated constant and its standard error (of -7 and 4.2) are large. This is always a possibility. In the present case the lack of significance would justify the use of regression through the origin. Precision is increased but accuracy is still relatively poor.

Along with strategy 1, strategy 2 leads to the most precise and least accurate estimates. This is attributable to the influence of the high proportion of large pieces in the sample. Attempting to discount this effect by measuring, but not subsequently using, the 10 largest pieces while retaining the same overall sample size, as in strategy 4, improves accuracy but estimates are less precise. Sampling evenly over the full range of the data, as in strategy 3, produced the most accurate results but with less precision than some of the other methods.

Some of these effects arise from the underlying mathematics of regression. In selecting a sampling strategy for other assemblages the following points must be borne in mind:

- using lots of large pieces can lead to increased precision; but
- if their behaviour is at variance with the theoretical model, or with the unmeasured pieces, then poor results may arise; nevertheless
- since the large pieces make the most important contributions to total area it may be sensible to over-sample from them, however they are subsequently used.

Conflicting demands thus arise in selecting a strategy and there is no point in being prescriptive. The potentially 'best' strategies may also be potentially 'worst' if things do not go to plan. Any assemblage needs to be treated on its merits. An illustration of this now follows.

20.7 An application

The lessons learned from the foregoing analyses were applied 'in anger' to an assemblage of Roman glass from domestic sites at Culver Street, Colchester. Weights and thicknesses were obtained for 117 specimens. The distribution of W/Z was rather different from the Catterick data with only 9 specimens having a value greater than 6. All these were sampled, together with a further 13 specimens sampled systematically (according to the W/Z ratio). This is a mixture of strategies 3 and 4 and is 'safe' rather than justified by theoretical considerations.

Data Set	n	$\hat{\alpha}$	$SE(\hat{\alpha})$	$\hat{\beta}$	$SE(\hat{\beta})$	$\hat{\gamma}$	$SE(\hat{\gamma})$
A	151	.16	(.21)	3.48	(.07)	1.43	(.13)
A	151	-	-	3.52	(.05)	1.38	(.13)
B	107	.89	(.39)	3.11	(.13)	1.17	(.26)
B	107	-	-	3.42	(.06)	.98	(.26)
C	44	-5.08	(1.09)	4.00	(.11)	1.43	(.35)
C	44	-	-	3.52	(.07)	1.76	(.35)

Table 20.1: Estimates of the Model and its Variance Structure, — Catterick. Data set A is the full data set; B and C are the subsets determined by whether or not W/Z is less than 6.

Strategy	Model	$\hat{T}$	SE($\hat{T}$)	($\hat{T} - T$)
1	1	2904	445	-416
1	2	3514	50	194
2	1	3573	71	253
2	2	3544	57	224
3	1	3405	101	85
3	2	3418	101	98
4	1	3499	117	179
4	2	3488	95	168

Table 20.2: Estimates of Total Area for Different Sampling Strategies — Catterick. For sampling strategies see the text. Weighted least squares with weights proportional to W/Z was used. Model 1 includes a constant term, 2 does not; the constant term was not significant at 5% in each case.

The original intention was to estimate the regression using weighted least squares, with $\gamma = 1$, for the sub-sample of 13. The data dictated otherwise.

A plot using all the data suggested a good linear relationship with a correlation of 0.98. Using MINITAB to fit a regression, the use of residual plots and diagnostic statistics revealed no unusual features apart from one unusually large piece (obvious graphically). The estimate of α was close to significance at the 5% level so that its omission from (or inclusion in) the model was not obviously justified. In particular there was absolutely no evidence of non-constant variance (a feature confirmed by using Aitkin's (1987) approach which gave $\hat{\gamma} = 0.4$ with standard error 0.6).

In the end, and in the interests of gaining further insight, models were fitted for the full data set and the subset of 13 only, with and without a constant and using unweighted and weighted least squares ($\gamma = 1$). Results are given in table 20.3.

The maximum difference between estimates corresponds to a piece of about 9 by 9 cm. so that, for this data set, the choice of method (among those used) is not critical. The estimated total area is close to 1200 square cms. with the most deviant results from this, those for unweighted regression through the origin, differing by about 50. Had a choice of a single method been forced on us after the initial data examination it would probably have been to use unweighted regression with a constant using the subset of 13.

20.8 Conclusions

The regression approach is a quick and flexible way of obtaining useful estimates of the total area contained in a large assemblage of window glass. The approach is made possible by the existence of interactive, user-friendly packages such as MINITAB and can exploit the availability of diagnostic methods for regression. Access to this or some similar package is needed, together with some familiarity with simple linear regression.

A possible broad strategy for analysis is :

1. Examine the distribution of W/Z.
2. Determine, on the basis of 1, a sampling strategy. Exhaustive sampling of unusually large pieces (if feasible) and systematic sampling of smaller pieces (with $W/Z < 6$ and ordered by the value of W/Z) seems safe and sensible. With lots of large pieces, over-sampling at the top end and systematic sampling elsewhere seems sensible.
3. Keep the sample size small. About 20 has been used (arbitrarily) for the data sets examined here. Provided the glass is to hand while the analysis is being done more pieces can be measured if the initial regression analysis so dictates.
4. Examine the data, initially, using standard regression methods. Identify clearly unusual pieces, if any, and remove them from the analysis. Decide on whether or not to use regression through the origin; whether or not to estimate separate regressions; choose weights *etc.*
5. Either apply the chosen strategy or, if a choice is not clear-cut, apply those methods that are competitive.

Sample	Method	$\hat{\alpha}$	$SE(\hat{\alpha})$	$\hat{\beta}$	$SE(\hat{\beta})$	$\hat{T}$	$SE(\hat{T})$
A	1	1.33	(.68)	3.08	(.11)	1229	48
A	1			3.26	(.07)	1146	24
A	2	1.02	(.50)	3.14	(.13)	1214	35
A	2			3.34	(.09)	1165	27
B	1	1.76	(.91)	2.80	(.31)	1188	43
B	1			3.33	(.16)	1147	42
B	2	1.21	(.83)	3.01	(.37)	1187	47
B	2			3.48	(.19)	1183	49

Table 20.3: Model and Total Estimates — Culver Street. Sample A uses all the 22 sampled observations, B the smallest 13 of these. Model 1 is unweighted and 2 weighted least squares, with weights proportional to W/Z.

In the latter case it is to be hoped that similar results are obtained; if not then there are likely to be internal clues (such as the size of the standard errors) as to which method(s) is (are) most useful.

In the absence of sophisticated aids for measuring area the potential time savings are considerable. As an example, for the Catterick data, measuring 20 rather than 151 pieces represents an obvious reduction in time. Given the creation of a data file with the necessary information in it the fitting of one or more regression models can be very rapidly accomplished. It is also straightforward to pull out information for particular contexts and estimate it separately, should this be of interest. Calculation of the estimated total area and its standard error is easy, given that the package used produces the required information, and facilitated by the use of macros with a package such as MINITAB (these are available from the first author).

This paper is a report on work in progress, on a problem that has rarely been addressed before. Two data sets have been examined and it would be clearly inappropriate to draw any definitive conclusions from this. The regression methodology proposed can be applied in a straightforward manner and requires the intelligent interaction of the analyst with the output obtained. With further experience it is hoped that a set of guidelines for such analyses, capable of being used by a finds analyst with access to a suitable regression package, can be produced.

20.9 Acknowledgements

The window glass from Catterick and Culver Street was made available to the Romano-British Glass Project by the Central Excavation Unit and the Colchester Archaeological Trust respectively. The Romano-British Glass Project is funded by the Historic Buildings and Monuments Commission (England).

Bibliography

AITKIN, M. 1987. "Modelling variance heterogeneity in normal regression using GLIM", *Applied Statistics*, 36: 332–339.

ATKINSON, A. C. 1985. *Plots, Transformations, and Regression*. Clarendon Press, Oxford.

BOON, G. C. 1966. "Roman window glass from Wales", *Journal of Glass Studies*, VII: 41–45.

CHARLESWORTH, D. 1959. "The glass in The Roman bath house at Red House, Beaufront near Corbridge", *Archaeologia Aeliana series 4*, XXXVII: 85–176.

CHARLESWORTH, D. 1972. "The glass", *in* Frere, S. S., (ed.), *Verulamium I*, RRCSAL 28, pp. 196–215. Society of Antiquaries of London, London.

CHARLESWORTH, D. 1979. "Glass (including material from all other Exeter sites excavated between 1971 and 1976)", *in* Bidwell, P. T., (ed.), *The legionary bath-house and basilica and forum at Exeter*, Exeter Archaeological Report I, pp. 222–231. Exeter City Council and University of Exeter, Exeter.

COLE, H. 1966. "Analyses and discussion of the Caerleon window glass", *Journal of Glass Studies*, VII: 46–47.

HARDEN, D. B. 1974. "Window glass from the Romano-British bath-house at Garden Hill, Hartfield, Sussex", *Antiquaries Journal*, LIV: 280–281.

HARDEN, D. B. & J. PRICE 1971. "The glass", *in* Cunliffe, B., (ed.), *Excavations at Fishbourne 1961-1969*, RRCSAL XXVII, pp. 317–368. Society of Antiquaries of London, London.

PRICE, J. 1977. "Trade in Glass", *in* du Plat Taylor, J. & Cleere, H., (eds.), *Roman shipping and trade: Britain and the Rhine Provinces*, CBA Research Report 24, pp. 70–78. Council for British Archaeology, London.

ROYALL, R. M. 1970. "On finite population sampling theory under certain linear regression models", *Biometrika*, 57: 377–387.

ZIENKIEWICZ, J. D. 1986. *The Legionary fortress baths at Caerleon. Volume I: the buildings*. National Museum of Wales and CADW, Cardiff.